Breaking HuFu with 0 Leakage

A Side-Channel Analysis

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Introduction

What is HuFu?

- Signature scheme based on unstructured lattices
- Based on the Hash-and-Sign paradigm [GPV08] (like Falcon)
- Round 1 candidate to NIST on-ramp post-quantum signature competition

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Why attack it?

- Absence of structure counters attacks on Falcon
- Trapdoor sampling a la [MP12] is used in other contexts (IBEs...)

Results

- We target sensible multiplications and the base discrete Gaussian sampler with power analysis and recover many coefficients of the signing key.
- The attacks are completed using lattice reduction whose cost we estimate depending on the amount of recovered coefficients

1. The HuFu Signature Scheme

Hash-and-sign for Lattices and HuFu

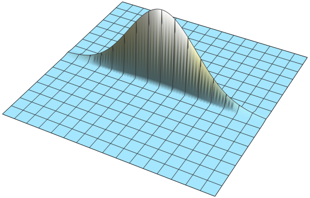
Generic framework for lattice-based signatures [GPV08] such as Falcon. Instanciated as follows for HuFu:

- Verification key: a matrix $\mathbf{A} = (\mathbf{I}_m|\hat{\mathbf{A}}|\mathbf{B})$ with $\mathbf{B} = p\mathbf{I}_m - \hat{\mathbf{A}}\mathbf{S} - \mathbf{E} \bmod pq$,
- Signing key: $\mathbf{sk}^\top = q(\mathbf{I}_m|\mathbf{S}|\mathbf{E})$, a short basis of $\Lambda = \{\mathbf{Ax} = 0 \bmod pq, \mathbf{x} \in \mathbb{Z}^k\}$,
- Given a message μ, sign by giving a short preimage $\mathbf{x}$ of $\mathbf{u} = H(\mu)$ by $\mathbf{A}$,
- How is $\mathbf{x}$ sampled?

First Try

Take $\mathbf{z} \hookleftarrow D_{\mathbb{Z}^k+\mathbf{v}/q,\bar{r}^2}$ and set

$$\mathbf{x} = \mathbf{sk} \cdot \mathbf{z}.$$



$$\mathbf{A} \cdot \mathbf{sk} \cdot \mathbf{z} = p\mathbf{v} \bmod pq$$

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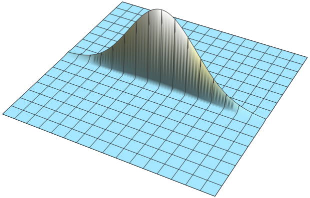
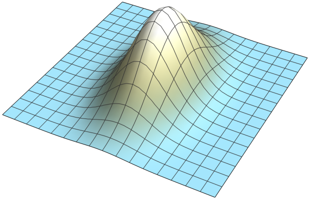


$$\mathbf{A} \cdot \mathbf{sk} \cdot \mathbf{z} = p\mathbf{v} \bmod pq$$

- Set $\mathbf{v} = \lfloor \mathbf{u}/p \rceil$: approximate preimage
- Add $\mathbf{u} \bmod p$ to get an exact preimage
- The distribution leaks sk!

Adding a Perturbation



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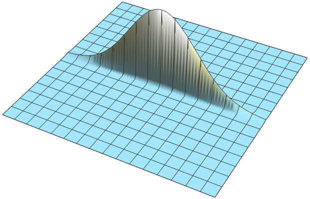

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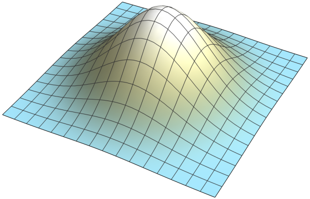

$\mathbf{p} \hookleftarrow D_{\mathbb{Z}^k,\Sigma_p}$

Sampled using Cholesky decomposition

$\mathbf{sk} \cdot \mathbf{z}$

$\mathbf{z} \hookleftarrow D_{\mathbb{Z}^k+\mathbf{c},\bar{r}^2}$

$\mathbf{c} = \lfloor(\mathbf{u} - \mathbf{Ap})/p\rceil/q$

$\mathbf{x}$

Short approximate preimage of $\mathbf{u}$

Not leaky

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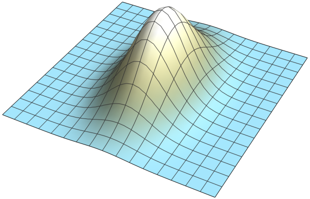

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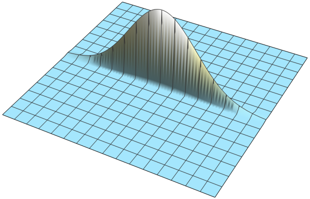

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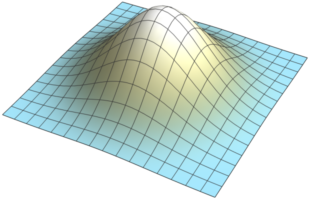

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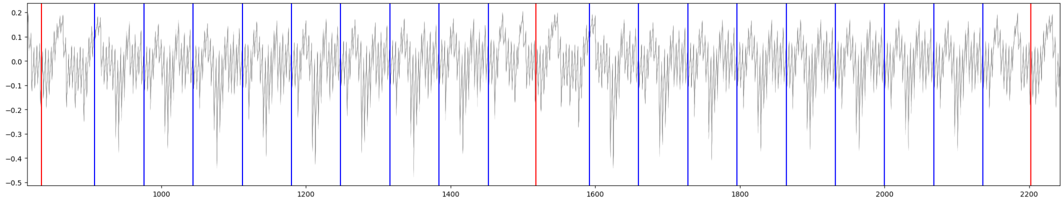
$\mathbf{c} = \lfloor (\mathbf{u} - \mathbf{A}\mathbf{p})/p \rceil / q$

$\mathbf{x}$
Short approximate preimage of $\mathbf{u}$
Not leaky

2. Side-Channel Analysis

Experimental set-up

Acquisition device: **ChipWhisperer Lite** with a Cortex M4 target.
Targetted C code is taken from the NIST submission package.



Code & some power traces available on a GitHub repository (link in paper).

Feel free to reach out!

Overview of the leakage spots

Algorithm HuFu Sign

1: $\mathbf{p} \leftarrow \text{SampleP(sk)}$
2: $(\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2) \leftarrow \mathbf{p}$
3: $\mathbf{v} \leftarrow \text{ComputeV}(\mathbf{A}, \mathbf{p}, \mu)$
4: $\mathbf{z} \leftarrow q \cdot \text{SampleZ}_d(\mathbf{v}/q)$ — Gaussian sampler
5: $\mathbf{x}_0 \leftarrow \mathbf{E}\mathbf{z} + \mathbf{p}_0$ — matrix-vector multiplication
6: $\mathbf{x}_1 \leftarrow \mathbf{S}\mathbf{z} + \mathbf{p}_1$ — matrix-vector multiplication
7: $\mathbf{x}_2 \leftarrow \mathbf{z} + \mathbf{p}_2$
8: **if** $\|(\mathbf{x}_0 + \mathbf{e}, \mathbf{x}_1, \mathbf{x}_2)\| > B$ **then**
9: goto 1
10: **end if**
11: **return** $\sigma = (\mathbf{x}_1, \mathbf{x}_2)$

Leakage in matrix-vector multiplication

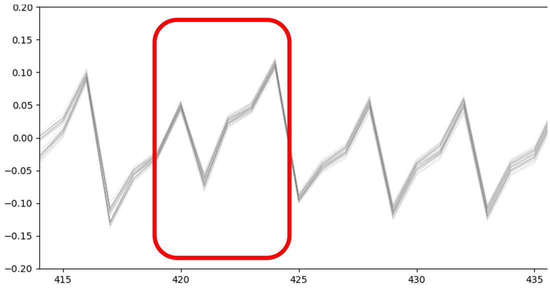
Targeted operations: $S_{i,j} \cdot z_i$ (resp. $E_{i,j} \cdot z_i$)

Coefficients of **S** (resp. **E**) are ternary and follow a binomial distribution.
$\rightarrow$ only three possible outputs for $S_{i,j} \cdot z_i$:

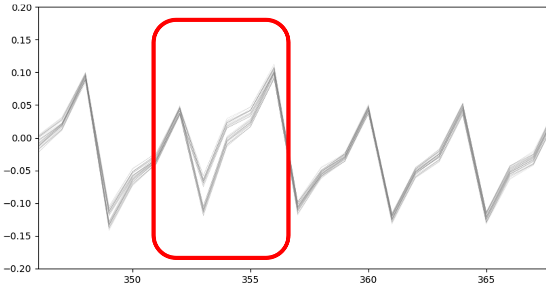
1. 0 (with probability 0.5)
2. z_i
3. $-z_i$

$\rightarrow$ we should see it in the power traces!

How to gain 0 leakage



(a) $S_{i,j} = 0$.



(b) $S_{i,j} \neq 0$.

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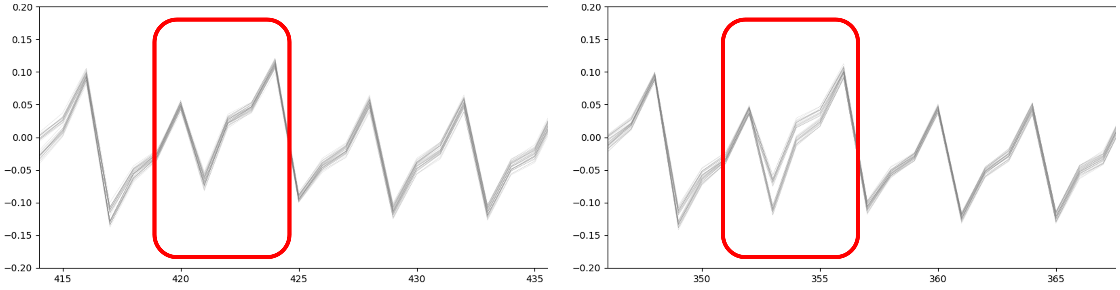

(a) $S_{i,j} = 0$. **(b)** $S_{i,j} \neq 0$.

With 1,500 traces, we can recover 98% of the $S_{i,j}$ (resp. $E_{i,j}$) equal to zero.

A (simple) countermeasure

$\mathbf{x}_0$ is used only in the following (non-sensitive) check:

$$\|(\mathbf{x}_0 + \mathbf{e}, \mathbf{x}_1, \mathbf{x}_2)\| > B$$

$\mathbf{x}_0 + \mathbf{e}$ can also be computed as follows:

$$\mathbf{x}_0 + \mathbf{e} = \mathbf{u} - \hat{\mathbf{A}}\mathbf{x}_1 - \mathbf{B}\mathbf{x}_2$$

which totally removes the secret component $\mathbf{E}$.

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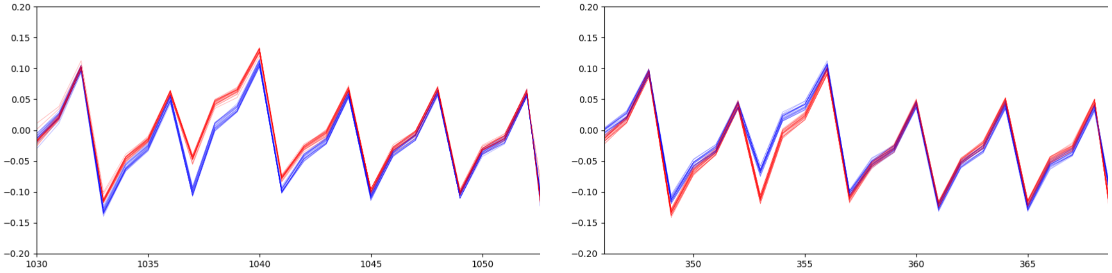
which totally removes the secret component $\mathbf{E}$.

$\rightarrow$ let's improve our attack to gain additional information on $\mathbf{S}$!

How to gain more-than-0 leakage

What if we had (by any chance) the sign of z_i?



(a) $S_{i,j} = -1$. **(b)** $S_{i,j} = 1$.

Figure: Power traces in red (resp. blue) correspond to $z_i < 0$ (resp. $z_i > 0$).

Leakage in Gaussian sampler

SampleZ(`center`):

1. $v \leftarrow \texttt{Rnd}(72)$
2. $c \leftarrow (\texttt{center} > 8) * (16 - 2 * \texttt{center}) + \texttt{center}$
3. $z^+ \leftarrow 0$
4. **for** $i = 0 \ldots 26$ **do**
5. $\quad z^+ \leftarrow z^+ + [\![v < \texttt{RCDT}[c][i]]\!]$
6. **end**
7. $z \leftarrow [\![\texttt{center} > 8]\!] * (27 - 2 * z^+) + z^+ - 13$
8. **return** z

input: $\texttt{center} \in [0, 15]$
output: $z \in [-12, 12]$

Consequences on the attack

$z = 0 \implies z^+ \in [13, 14]$ (depending on `center` value)
This implies 13 or 14 incrementations in the for loop.

Previous attacks on other schemes were relying on the fact that

$$z = 0 \iff \text{no incrementation at all}$$

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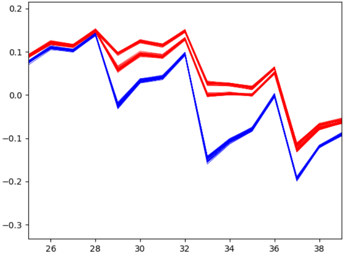
Now we have much more noise!

$\rightarrow$ we will not target the for loop

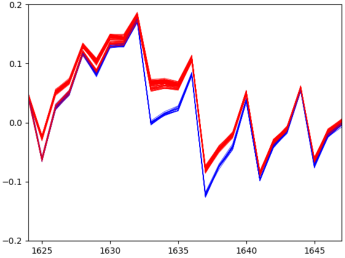
Targetted C code

```
int c = center;
c = (c > 8) * (16 - 2 * c) + c;                    // c computation
z = 0;
for (u = 0; u < TABLE_LEN; u += 3)
{
    uint32_t w0, w1, w2, cc;
    w0 = dist0[c][u + 2];
    w1 = dist0[c][u + 1];
    w2 = dist0[c][u + 0];
    cc = (v0 - w0) >> 31;
    cc = (v1 - w1 - cc) >> 31;
    cc = (v2 - w2 - cc) >> 31;
    z += (int)cc;
}
return (center > 8) * (27 - 2 * z) + z - 13;       // z computation
```

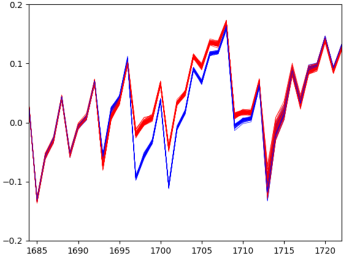
Sign recovery of z



(a) Computation of c.



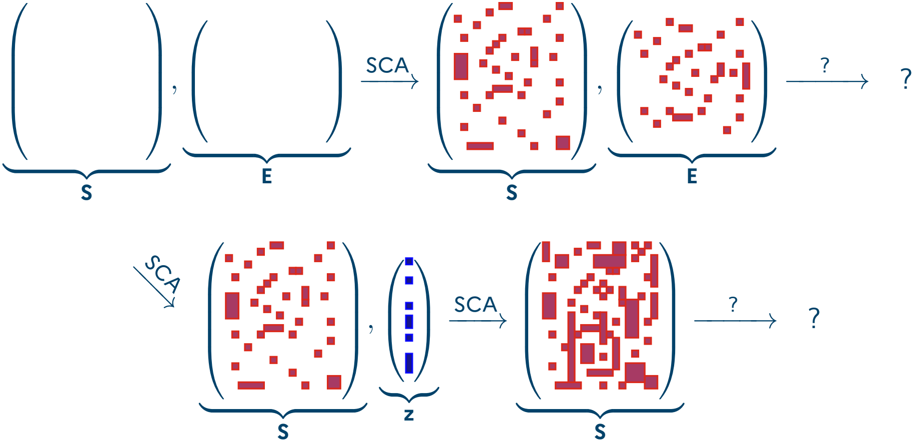
(b) First multiplication.



(c) Final subtraction.

With 1,500 traces, we can recover 75% of the $S_{i,j}$ given prior information on z_i.

Attacks Summary



3. Forgery

First attack when S and E are known

Given an LWE sample $\mathbf{As} + \mathbf{e}$ and some 0s of $\mathbf{s}$ and $\mathbf{e}$, how do we exploit them?

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First attack when S and E are known

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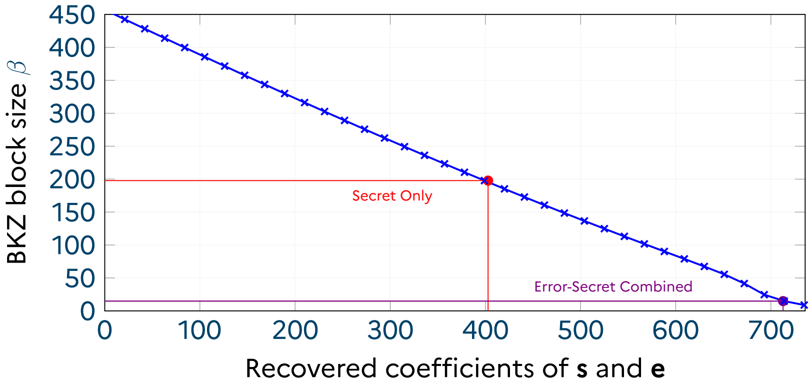
- Remove the i-th column of $\mathbf{A}$ if $s_i = 0$: dimension reduced by one.
- Write $b_i = \langle \mathbf{a}_i, \mathbf{s} \rangle$ if $e_i = 0$. Dimension reduced by one. Some rewriting involved to find a new LWE instance with one less dimension.

What is the cost of BKZ on the new LWE instance once every hint has been incorporated?

Remaining Cost of the Attack



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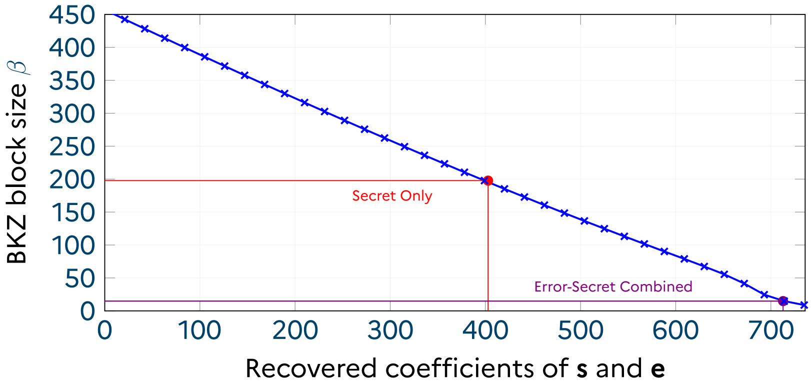

Conclusion: preventing the leakage on **E** is critical.

Forging for specific vectors

Assuming the first k columns $\mathbf{S}_k$ of $\mathbf{S}$ are known via the previous attack, what can we do with them?

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- If the target is $\mathbf{u} = \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{0} \end{pmatrix}$, then we set $\mathbf{p} = \mathbf{0}$, $\mathbf{v} = \lfloor \mathbf{u}/p \rceil$ and $\mathbf{z} = \mathbf{v}$. A signature would then be:

$$\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{S} \\ \mathbf{I}_m \end{pmatrix} \cdot \mathbf{v} = \begin{pmatrix} \mathbf{S}_k & \mathbf{0} \\ \mathbf{I}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \cdot \mathbf{v}.$$

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This vector is short, but which message did we sign?

Finding specific vectors

- Choose any μ and compute $\mathbf{u} = H(\mu) = \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{pmatrix}$.
- Write $\mathbf{A} = \begin{pmatrix} \mathbf{A}_h \\ \mathbf{A}_l \end{pmatrix}$
- Find short $\mathbf{x}'$ such that $\mathbf{A}_l \mathbf{x}' = \mathbf{u}_2$ with lattice reduction
- Set $\mathbf{u}' = \mathbf{u} - \mathbf{A}\mathbf{x}' = \begin{pmatrix} \mathbf{u}'_1 \\ \mathbf{0} \end{pmatrix}$
- We are back to the previous case!

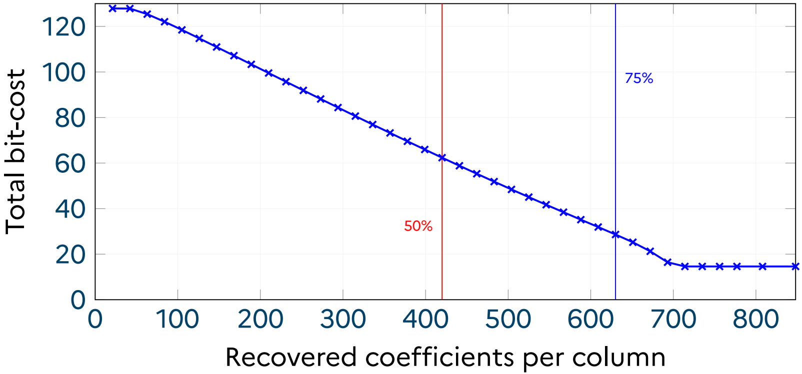
How much costs a forgery?

We start by gathering d coefficients per column.

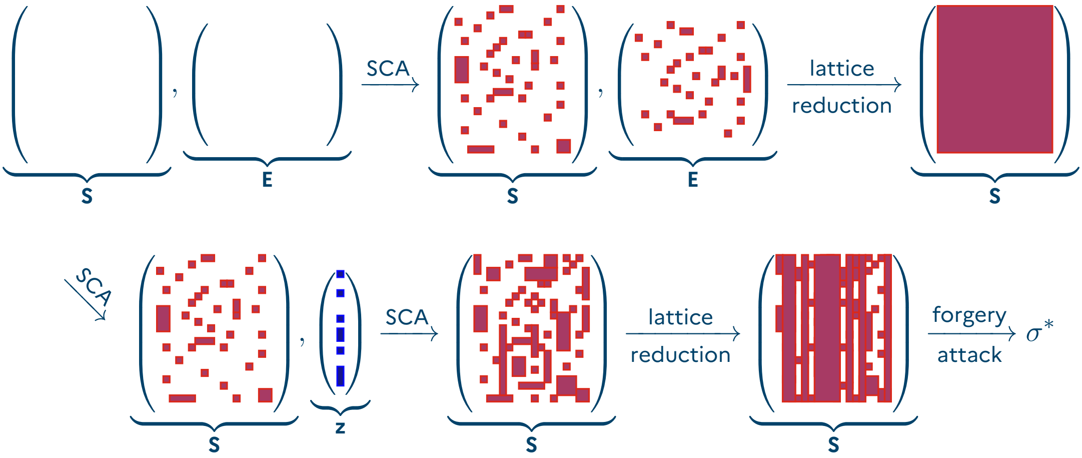
- First step: complete k columns via lattice reduction: k times LWE with dimension reduced by d
- Second step: one more lattice reduction to find $\mathbf{x}'$: dimension reduced by k but bound B' on $\|\mathbf{x}'\|$ that worsens with k
- Third step: forgery for specific vectors (essentially free)

All that remains is to optimize over k.

Final Cost


Total bit-cost
120
100
80
60
40
20
0
0
100
200
300
400
500
600
700
800
Recovered coefficients per column
50%
75%

Attacks Summary



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Our approach is flexible:

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Thank you for your attention!